ANALYSIS AND BEHAVIOR OF FRP COMPOSITES FOR NEW AND EXISTING STRUCTURES

Ibrahim M. Ibrahim* Ibrahim G. Shaaban**

ABSTRACT

Although fiber reinforced plastics (FRP) are now widely used in a range of structural applications, e.g air crafts, cars.....etc, they are still rarely used by civil engineers. The main reasons for this are the complexities arising from the nature and structure of these materials which affect their analysis and design procedures. This paper describes a non linear static and dynamic finite element analysis package (NSDFEM) suitable for the analysis of all types of structures made of FRP. The package is also capable of modeling reinforced concrete structures retrofitted with FRP.

1 INTRODUCTION

Advanced composites or specifically Fiber reinforced plastics (FRP) laminates are being increasingly used in structural applications. FRP composites are generally made of filaments (3 to 5 microns in diameter) bonded together using a resin matrix. The mechanical properties of. composites vary significantly with the type and orientation of the constituent resins and fibers [1]. The recent advancements in the field of advanced composites showed that these new materials have good potential for application in new construction or in the repair of concrete structures in areas where conventional materials fail to produce satisfactory performance and/or are known to have limited service life [2]. To date, however, advanced composites have been rarely used in civil engineering structures. The reason for this has been mainly their high cost and lack of familiarity of civil engineers with their properties, manufacturing techniques and methods of analysis [3]. The high initial cost of FRP materials is offseted by their maintenance free performance which also includes the economic, social or even political costs when deterioration of conventional structures occur.

The analysis of FRP sections is normally based on the layered approach which deals with the FRP as laminated composites where each lamina has a different orientation of fibers [4]. Finite element programs can be used for predicting the response of structures made of FRP sections [5 and 6]. However, each of these programs is limited to a specific problem. There is a need for an accurate and general package for the analysis and design of the structures made or retrofitted with FRP.

Professor and Vice Dean for Postgraduate Studies, Faculty of Engineering, Shoubra.
Assistant Professor, Civil Engineering Dept., Faculty of Engineering, Shoubra.

The main objective of this paper is to present a non-linear static and dynamic finite element analysis technique (NSDFEM) capable of analyzing frames, slabs and shells made with FRP. This technique can also model the cases which utilize FRP composites for the rehabilitation and retrofitting of the concrete structures. The cases under consideration involve flexure and shear strengthening due to cracking of concrete or for increasing the load carrying capacities of the structure, as well as the cases where FRP composites are used as reinforcement.

For the purpose of strengthening reinforced concrete structures, the FRP laminates are attached to the structure in question by epoxy resin (see Figure 1). The whole section is considered to be a layered system representing the combined responses of the non-linear behavior of concrete in compression, concrete in tension, as well as reinforcing steel. The analysis of such laminated construction is carried out using an efficient direct iterative procedure which incorporates the material non-linearity, cracking, tension stiffening and compression softening of concrete. In order to predict both of static and dynamic responses of the structure accurately, a realistic mathematical modeling of the material behavior, as well as rational assumptions, were adopted.

2 THEORETICAL FORMULATION FOR FRP LAMINATES

2.1 Force Deformation Relationship

The thin plate shown in Figure 2 (a) is considered to be subjected to inplane forces N_x, N_y, N_{xy}, and to moments M_x, M_y, M_{xy}. The forces and moments are stress and moment resultants specified per unit length of the plate. It is assumed that the plate is composed of a stack of anisotropic laminas fully bonded at their interfaces as shown in Figure 2 (b). Since the laminas have different properties, the middle surface of the plate is not an axis of symmetry. According to these assumptions, the force-deformations relationship may be written as

$$\begin{bmatrix} (N) \\ (M) \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{bmatrix} (\varepsilon_o) \\ (K) \end{bmatrix} \qquad (1)$$

where $\{\varepsilon_o\}$ are the total extentional and shear strains of the middle plane of the plate, and $\{K\}$ is the vector of its bending and twisting curvatures. The matrices [A], [D] and [B] are the membrane, bending and coupling stiffness matrices. The elements of these matrices depend on the geometric and material properties of the laminas [7].

element
th FRP.
bilitation
xure and
apacities

ite
ection is
on-linear
eel. The
terative
ng and
sponses
as well

N_x, N_y,
moment
osed of a
). Since
axis of
may be

and {K}
] are the
epend on

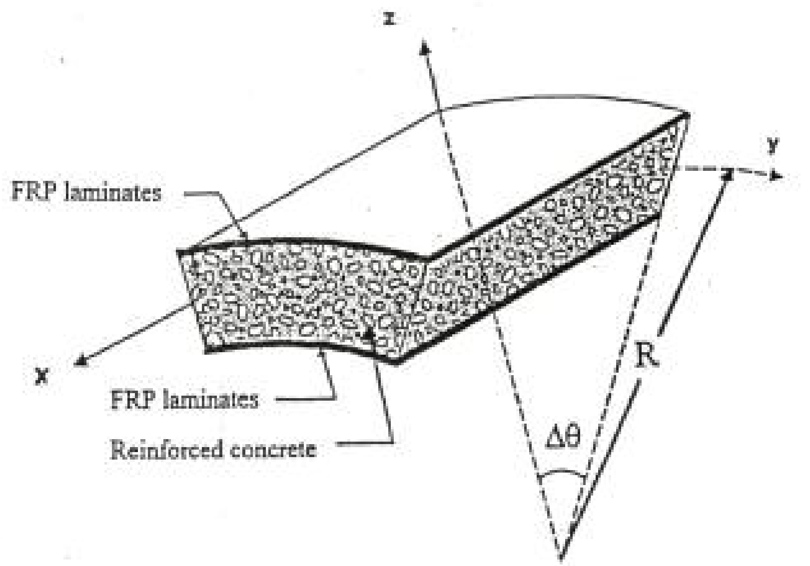
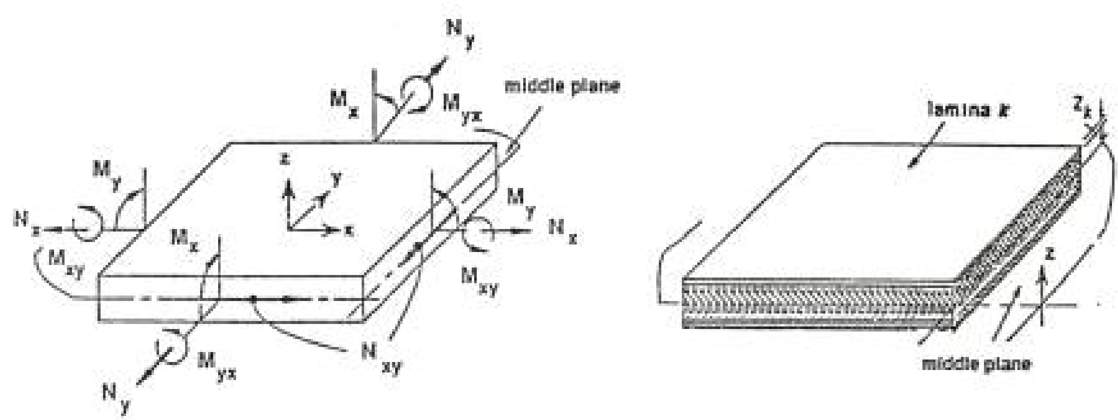


Figure 1 Reinforced concrete shell segment retrofitted with FRP laminates



(a) Stress resultants

(b) Laminas stacking

Figure 2 Typical laminated element

For a composite plate composed of n layers, with lamina k located at a distance z_k from the middle plane of the plate and assuming z to be positive upward (see Figure 2.b), the elements of matrices [A], [D] and [B] are given by

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left[Z_k - Z_{k-1}\right] \tag{2.a}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left[Z_k^2 - Z_{k-1}^2\right] \tag{2.b}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left[Z_k^3 - Z_{k-1}^3\right] \tag{2.c}$$

where (i = j = 1, 2, 6) and the quantities Q_{ij} are material properties. The full derivation of the individual elements of the [A], [D] and [B] matrices is detailed elsewhere [4].

Equations (1 and 2) are of a general nature and are applicable to bars, beams, panels, plates and shells made of FRP laminates. The matrix [B] implies that the membrane and bending actions in a general anisotropic FRP composites are coupled. It is evident that such coupling does not exist for sections made of isotropic materials [2].

2.2 Types of Laminated Systems

2.2.1 Symmetric Laminates

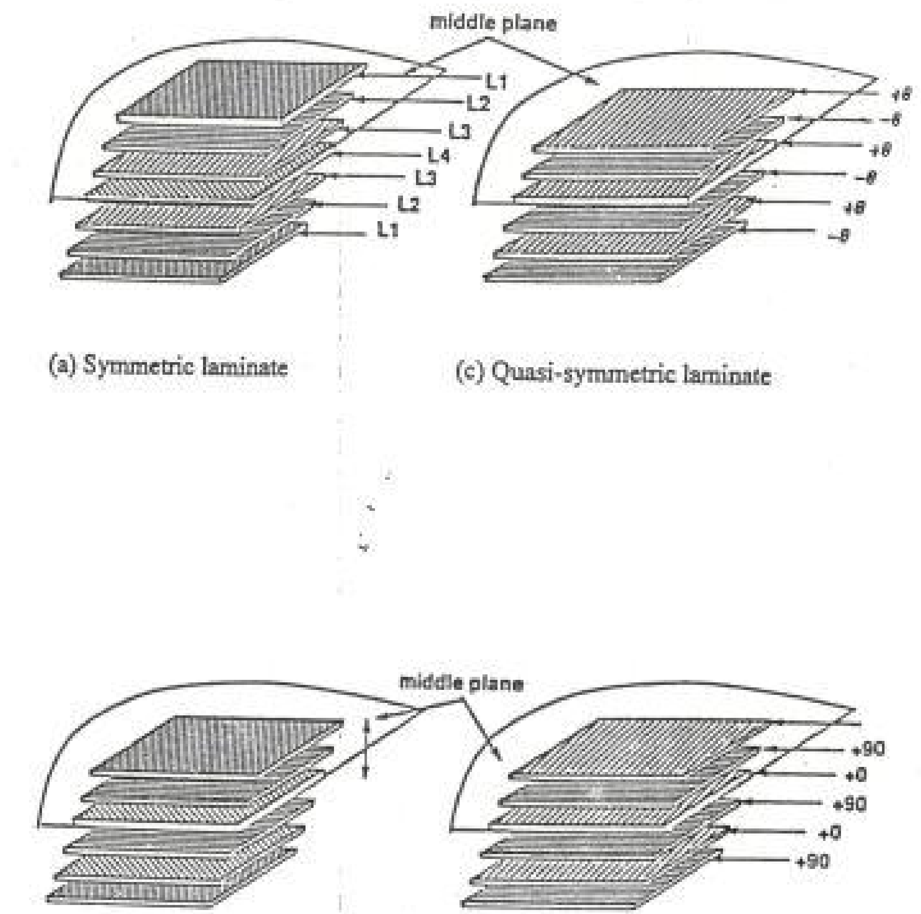
If a laminated system has identical lamina of the same thickness, orientation and position relative to the middle plane of the plate, then it is referred to as a symmetric laminate (Figure 3.a). For symmetric laminates, the middle plane is a plane of material symmetry and therefore sub-matrix [B] in Equation (1) is null which means that there is no bending-membrane coupling.

2.2.2 Balanced Laminates

In the balanced laminate for every lamina with fibers oriented at angle above the middle plane, there is a corresponding lamina of equal thickness and material below the middle plane with fiber orientation , but the position of the anti symmetric laminas from the middle plane is not the necessarily same (Figure 3.b). In a balanced laminated system, some terms of the membrane matrix [A] are zero and this means that there is no coupling between the inplane extensional and shear effects.

2.2.3 Quasi-Symmetric Laminates

It is a system in which there is symmetry with respect to thickness and position of the laminas but anti-symmetry with respect to their orientation (Figure 3.c). In such laminates some terms of the bending matrix [D] are zero which means that no coupling exists between the bending and twisting effects.



(a) Symmetric laminate

(c) Quasi-symmetric laminate

(b) Balanced laminate

(d) Cross-ply laminate

Figure 3 Different types of laminates

2.2.4 Cross-Ply Laminates

In these laminates, all the lamina are oriented at 0 or 90 degrees (see Figure 3.d). Due to this configuration, the bending-twisting coupling vanishes in this laminated system.

2.3 Strain Displacement Relations

For the thin plate element shown in Figure 2, it is assumed that it follows the classical thin plate behavior with small deformations [4]. The strain-displacement relations for this plate can be written as follows:

$$\{\varepsilon\} = \varepsilon_0 + z\,\{K\} \qquad (3)$$

Where $\varepsilon_o = \varepsilon_x^o, \varepsilon_y^o$ and γ_{xy}^o are the membrane $\left(\varepsilon_x^o, \varepsilon_y^o\right)$ strains and shear strain $\left(\gamma_{xy}^o\right)$

$K = K_x, K_y$ and k_{xy} are vectors for flexural and twisting curvatures;

in which

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} = \begin{bmatrix} \frac{\partial u_o}{\partial x} \\ \frac{\partial v_o}{\partial y} \quad \frac{\partial_{vo}}{\partial_y} \\ \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \end{bmatrix} ; \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ \frac{2\partial^2 w}{\partial x \partial y} \end{bmatrix} \qquad (4)$$

where u_o, v_o and w are the displacement components in the x, y and z directions and the superscript o refers to the middle plane.

Stresses σ_x, σ_y and τ_{xy} in a lamina can be determined from the known strains using

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{26} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \qquad (5)$$

where the derivation of the elements of matrix $[\overline{Q}]$ are given elsewhere [2]. From the design point of view, the stresses in the material coordinates are of greater interest because these can be compared with the allowable design stresses. Ashton et al [4] described the transformation of coordinates to the material coordinate system in detail.

3 STATIC AND DYNAMIC FINITE ELEMENT FORMULATION

3.1 Strain Energy of Anisotropic FRP Laminates

The strain energy of anisotropic laminates can be written as

$$U = \frac{1}{2}\int_{v} [\varepsilon]^{T} [\sigma]\, dv \qquad (6)$$

where v is the total volume of the system.

By substituting Equations 2,4 and 5 into Equation (6), the strain energy can be written in terms of the surface area as follows:

$$U = \frac{1}{2}\int_{s} \begin{bmatrix} [\varepsilon_0] \\ [K] \end{bmatrix}^{T} \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{bmatrix} [\varepsilon_0] \\ [K] \end{bmatrix} ds \qquad (7)$$

where s is the surface area of the laminated system.

Or

$$U = \frac{1}{2}\int_{s} [\varepsilon_0]^{T} [A][\varepsilon_0] + 2[\varepsilon_0][B][K] + [K]^{T}[D][K]\, ds \qquad (8)$$

The final form of Equation (8) can be obtained by substituting Equation (4) into Equation (8) [8].

3.2 Work Equivalent Loads

The potential of the applied loads W acting on an element is represented by

$$\begin{aligned} W = &\int_{S} \left[\bar{p}_x u + \bar{p}_y v + \bar{p}_z w\right] ds \\ &+ \oint_{y} \left[\bar{N}_x u + \bar{N}_{xy} v + \bar{Q}_{xz} w - \bar{M}_x w_x - \bar{M}_{xy}\left(w_y - \frac{v}{R}\right)\right] dy \\ &- \oint_{x} \left[\bar{N}_{yx} u + \bar{N}_y v + \bar{Q}_{yz} w - \bar{M}_{yx} w_x - \bar{M}_y\left(w_y - \frac{v}{R}\right)\right] dx \end{aligned} \qquad (9)$$

where $\bar{p}_x$, $\bar{p}_y$, $\bar{p}_z$ are applied reference surface tractions in the x , y and z directions respectively. The applied forces $\bar{N}_x$, $\bar{Q}_{xz}$ etc. and moments $\bar{M}_x$, $\bar{M}_y$ etc. are the components acting on the edges of the layers and are positive when they act in the same direction as the force and moment resultants shown in Figure (2).

3.3 Kinetic Energy Formulations

The kinetic energy of a element can be expressed as

$$T=\frac{1}{2}\int_S \{Q\left[\dot{u}^2+\dot{v}^2+\dot{w}^2\right]-2J\left[\dot{u}\,\dot{w}_x-\dot{v}\left(\dot{w}_y-\frac{\dot{v}}{R}\right)\right]+I\left[\dot{w}_x^2+\left(\dot{w}_y-\frac{\dot{v}_y}{R}\right)^2\right]\}\,ds \qquad (10)$$

where differentiation with respect to time is represented by a dot. In Equation (10), Q, J, and I are inertia constants defined by

$$[Q,\ J,\ I]=\int_{-d_1}^{d_2}\left[1,\ z,\ z^2\right]\rho\ (z)\ dz \qquad (11)$$

$\rho(z)$ is the mass density of the element and may be a step function through the thickness in order to account for the transverse heterogeneity of the layers. Note that Q is associated with the translatory inertia terms and I with the rotary inertia contributions; J is associated with coupling between translatory and rotary inertia and vanishes for balanced laminated section for which $d=\frac{1}{2}t$.

3.4 Finite Element Discritization

(a) Beams and Frames

In order be show the effect of anisotropy in beams and frames, the governing uncoupled Equations (12 and 13)of a beam subjected to a lateral load y(x) and axial load P(x) is considered (see Figure 4).

$$\frac{d^4w}{dx^4}=-\frac{q(x)}{\overline{D}}-\frac{1}{\overline{B}}\frac{dp(x)}{dx} \qquad (12)$$

$$\frac{d^3u_0}{dx^3}=-\frac{q(x)}{\overline{B}}-\frac{1}{\overline{A}}\frac{dp(x)}{dx} \qquad (13)$$

where u_0 and w are axial and lateral displacements of the beam and x is a coordinate along the axis of the beam. $\overline{A}$, $\overline{B}$ and $\overline{D}$ are rigidities similar to those given by Equation (2). It can be seen that the governing equation of membrane is third order unlike the corresponding second order equation for isotropic beam. It can be noticed also that an axial force produces lateral displacement and vice versa. This is the coupling effect which was reported in sections 2.1 and 2.2.

For frame members , the stiffness matrix is based on a linear displacement function for extension and a cubic function for lateral displacement due to bending.

(10)

. Q, J, and I

(11)

through the
te that Q is
utions, J is
or balanced

uncoupled
oad. P(x) is

(12)

(13)

ong the axis
een that the
equation for
nt and vice

unction for

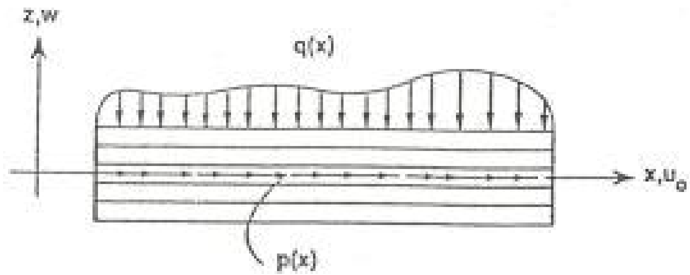
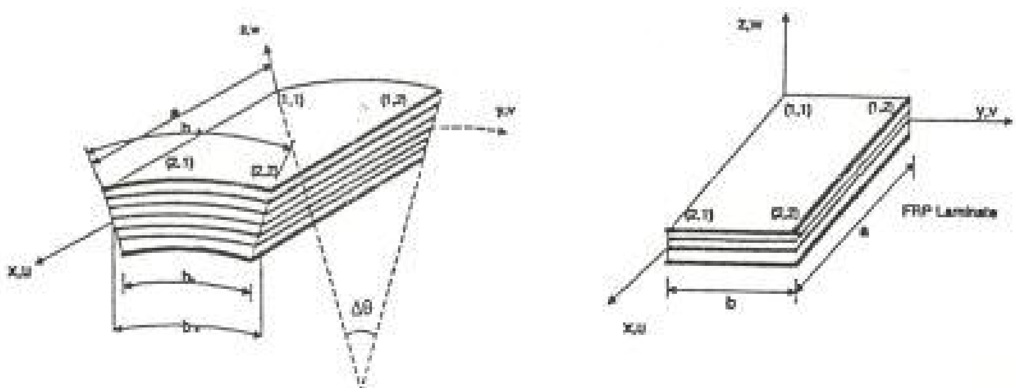


Figure 4 Typical laminated FRP beam



(a) Shell element

(b) Plate element

Figure 5 A high precision laminated element

(b) Plates and Shells

Most FRP laminated composite structures are in the form of panels, plates and shells.

The plate and cylindrical shell elements are shown in Figure (5). The displacement components u , v and w are approximated by the assumed displacement patterns. These assumed displacement patterns are represented by the sum of products of one-dimensional first-order interpolation polynomials and undetermined nodal coefficients. The reference surface displacement u of an FRP laminated cylindrical shell element is given as an example and is represented by:

$$u(x,y) = \sum_{i=1}^{2} \sum_{j=1}^{2} \left[H^{(1)}_{0i}(x)\, H^{(1)}_{0j}(y)\, u_{ij} + H^{(1)}_{1i}(x)\, H^{(1)}_{0j}(y)\, u_{xij} + H^{(1)}_{0i}(x)\, H^{(1)}_{1j}(y)\, u_{yij} + H^{(1)}_{1i}(x)\, H^{(1)}_{1j}(y)\, u_{xyij} \right] \qquad (14)$$

where , for example $u_{ij} = u\,(x = x_i,\ y = y_j = R\theta_j)$ are the nodal displacements and

$$u_{yij} = \frac{\partial u}{\partial y}(x = x_i, y = y_j = R\Theta_j) = \frac{1}{R}\frac{\partial}{\partial \Theta}(x = x_i, \Theta = \Theta_j) \qquad (15)$$

are the derivatives of u in the circumferential direction at the node points. Similar expressions are used for u , v, and w. and detailed elsewhere [8].

The $H^{(1)}_{Ki}(x)$ are the one-dimensional first-order interpolation polynomials given by

$$\begin{aligned} H^{(1)}_{01}(x) &= (2x^3 - 3ax^2 + a^3)/a^3; \quad H^{(1)}_{02}(x) = -(2x^3 - 3ax^2)/a^3 \\ H^{(1)}_{11}(x) &= (x^3 - 2ax^2 + a^2x)/a^2; \quad H^{(1)}_{12}(x) = (x^3 - ax^2)/a^2 \end{aligned} \qquad (16)$$

where a is the length of the element in the x direction. Corresponding expressions for the y direction are obtained by replacing x by y_s and a by b_s where

$y_s = y, \quad b_s = b$ for the plate

and

$y_s = R_s\Theta, b_s = R_s(\Delta\Theta)$ for the cylinder

The subscript s indicates that the lengths are measured on the appropriate reference surface.

4 LAMINATED IDEALIZATION OF RETROFITTING REINFORCED CONCRETE STRUCTURES

The analysis of reinforced concrete structures considering the material non-linearity of concrete can be carried out by assuming that the compression zone of a cross-section can be modeled by a number of layers (laminas), each lamina is assumed to have constant average stress distribution obtained from the stress- strain relationship of concrete in compression (Figure 6), and corresponding to the level of strain of the lamina under consideration. Subsequently, the elastic constant for each lamina in a given element under specified load level can be assumed to be the secant modulus at the corresponding level of strain of the lamina. The tension zone of the cross-section can also be modeled in a similar procedure considering the stress- strain relationships of concrete in tension, in addition to the tension stiffening of cracked concrete. Note that steel reinforcement can be considered as one of the laminas of the developed laminated construction. When the analysis is carried out for the cases of rehabilitation of reinforced concrete structures by FRP composites, such composite construction can also be considered as another lamina of the laminated system, (see Figure 7).

5- NON-LINEAR ANALYSIS FOR R.C STRUCTURES RETROFITTED WITH FRP LAMINATES

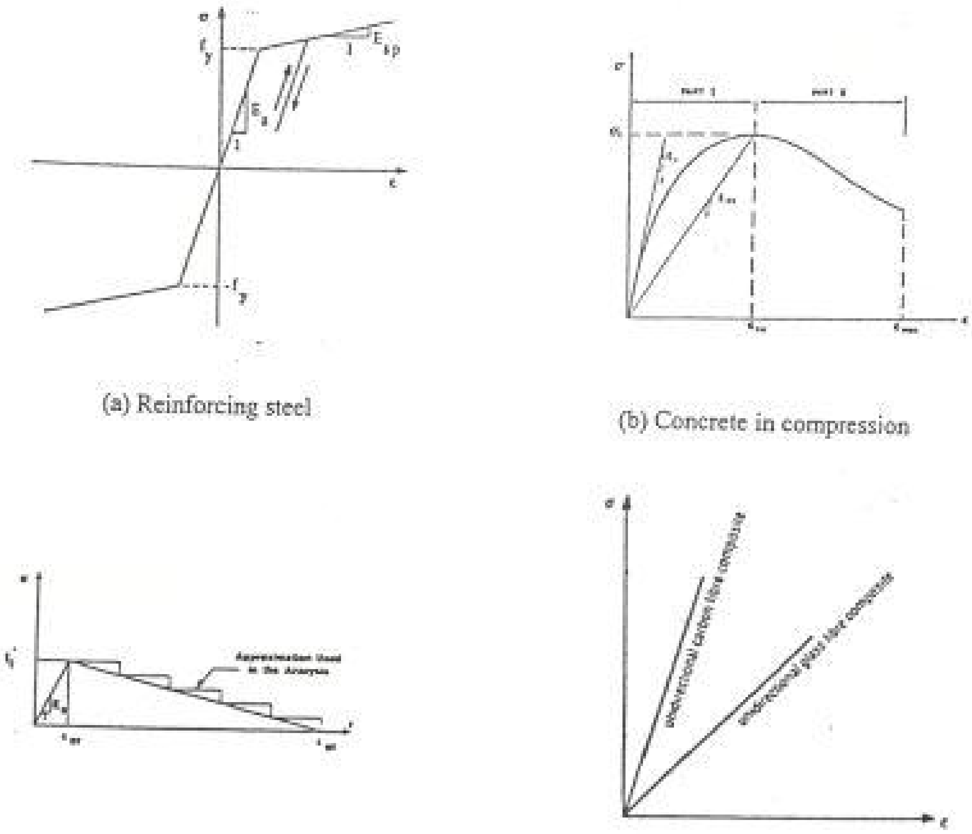
The nonlinear structural response of the retrofitted reinforced concrete structures in question is carried out using an efficient direct iterative procedure. A sample of the iterative method used in the non-linear analysis of strengthened R.C. frame is given by the flow chart shown in Figure (8).

6 CONCLUDING REMARKS

The **NSDFEM** finite element package was demonstrated to be developed especially for the analysis of anisotropic FRP laminates. The formulation for both FRP laminates and reinforced concrete sections strengthened with FRP was incorporated in the program. The frame, plate and shell elements used in the program were chosen to be high precision elements in order to predict the response of structures in question accurately. **NSDFEM** carries out nonlinear analysis problems incrementally using a direct iterative method.

It is clear that the heterogeneity of the section and anisotropy of the laminas have a very complex influence on the static and dynamic response of concrete structures strengthened with FRP. But with the aid of a numerical tool as **NSDFEM**, the analyst has the ability to conveniently and effectively quantify this interesting material and structural behavior.

It is hoped that in the future, FRP composites may become more widely used either on their own right or in the repair of existing structures.



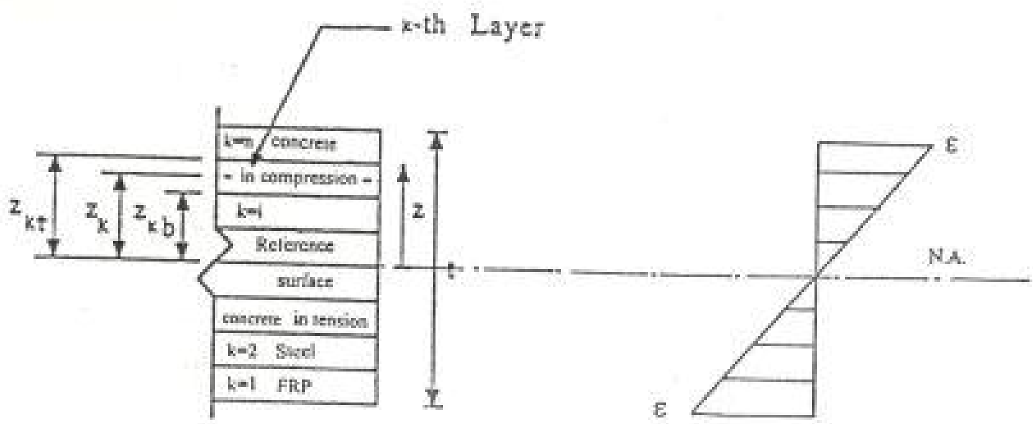
(a) Reinforcing steel

(b) Concrete in compression

(c) Concrete in tension

(d) FRP laminates

Figure 6 Idealized stress-strain relationships



(a) R.C and FRP in laminated form

(b) Strain distribution

Figure 7 Laminated idealisation of R.C element combined with FRP laminates

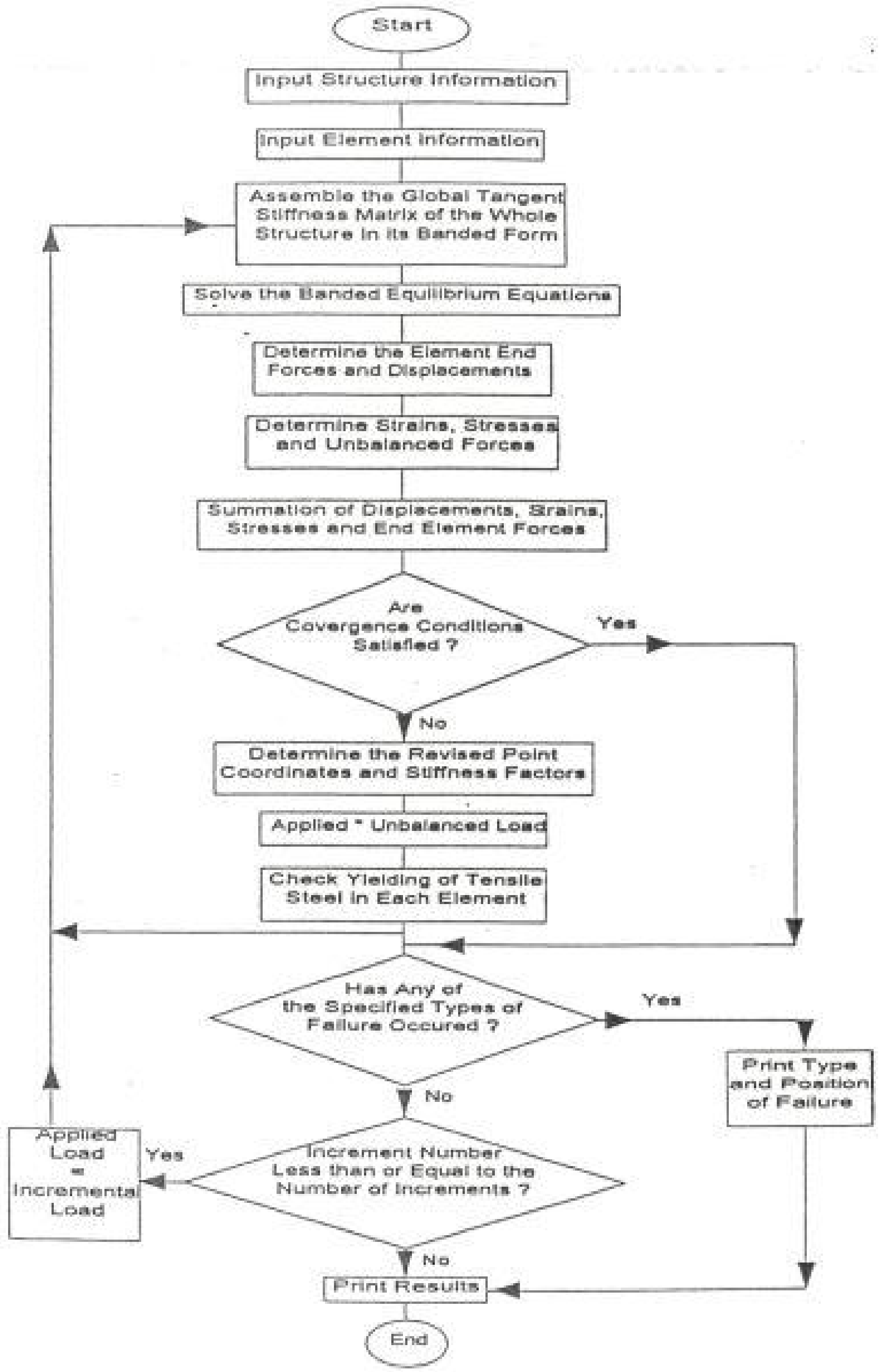


Figure 8 Flow Chart For the Plane Frame Program

7 REFERENCES

1 Saadatmanesh, H "Fiber Composites for New and Existing Structures" ACI Structural Journal, Vol. 91, No. 3, May-June 1994, pp. 346-354.

2 Razaqpur, A. G "Method of Analysis for Advanced Composite Structures" Advanced Composites Materials in Civil Engineering Structures, Ed. S.L. Iyer and R. Sen, ASCE, 1991, pp. 336-347.

3 Mallick, P. K "Fiber-reinforced composites: Materials, Manufacturing, and Design" Morcel Dekker Inc., New York, 1988, pp. 145-165.

4 Ashton, J. E, Halpin, J. C and Petit, P. H "Primer on Composite Materials" Technomic Publishing Co.; Stamford, Conn., 1969.

5 Shaaban, I. G and Ibrahim, I. M "Analysis of Ferrocement Roof Structures: New Construction and Utilization in Repair Procedures" The 4th International Conference on Concrete Engineering and Technology, Malaysia, 6-8 June 1995.

6 Ali, H. N and Shaaban, I. G "Analysis of Skew Composite Plates" CERM, Al Azhar University, Vol. 16, No. 7, July 1994, pp. 616-631.

7 Ibrahim, I. M and Monforton, G. R "Finite Strip Laminated Sandwich Roof Analysis" ASCE, Journal of the Structural Division, Vol. 105, No. ST5, May, 1979, pp. 905-919.

8 Monforton, G. R and Schmit, L. A "Finite Element Analysis of Sandwich Plates and Cylindrical Shells with Laminated Faces" Proceedings of the Conference on Matrix Methods in Structural Mechanics, AFFDL-TR-68-150, Wright-Patterson A.F.B., Ohio, 1969, pp. 573-616.